Σήματα και Συστήματα

DFT

Π. Ριζομυλιωτης

Παράδειγμα

- *Να υπολογιστεί η κυκλική συνέλιξη των ακολουθιών*

x[n]={1,0,2.5,1.5}

y[n]={1,1,0.5,2}

Λύση

Κατασκευάζω την κατοπτρική της $y_{(s)}(m) = y((-m))_N$

$$y_{(s)}(0) = y((0))_N = 1$$

$$y_{(s)}(1) = y((-1))_N = y[3] = 2$$

$$y_{(s)}(2) = y((-2))_N = y[2] = 0.5$$

$$y_{(s)}(3) = y((-3))_N = y[1] = 1$$

Παράδειγμα

Κατασκευάζω την κατοπτρική της

$$x \to \{1,0,2.5,1.5\}$$

$$y_{(s)} \to \{1,2,0.5,1\}$$

$$z[0] = 1\times 1 + 0\times 2 + 2.5\times 0.5 + 1\times 1.5 = 3.75$$

$$x \to \{1,0,2.5,1.5\}$$

$$y_{(s)}(m-1) \to \{1,1,2,0.5\}$$

$$z[1] = 1\times 1 + 0\times 1 + 2.5\times 2 + 1.5\times 0.5 = 6.75$$

$$x \to \{1,0,2.5,1.5\}$$

$$y_{(s)}(m-1) \to \{0.5,1,1,2\}$$

$$z[2] = 1\times 0.5 + 0\times 1 + 2.5\times 1 + 1.5\times 2 = 6$$

Παράδειγμα

Κατασκευάζω την κατοπτρική της

$$x \to \{1,0,2.5,1.5\}$$

$$y_{(s)} \to \{2,0.5,1,1\}$$

$$z[3] = 1\times 2 + 0\times 0.5 + 2.5\times 1 + 1\times 1 = 6$$

<u>Συνολικά</u>

$$z[0] = 3.75$$

$$z[1] = 6.75$$

$$z[2] = 6$$

$$z[3] = 6$$

Παράδειγμα

- *Να υπολογιστεί η γραμμική συνέλιξη των ακολουθιών*

x[n]={1,0,2.5,1.5}

y[n]={1,1,0.5,2}

Λύση

Κατασκευάζω τις επαυξημένες ακολουθίες μέχρι μήκος 2N

x_e[n]={1,0,2.5,1.5,0,0,0,0}

y_e [n]={1,1,0.5,2,0,0,0,0}

Υπολογίζω την κυκλική τους συνέλιξη. Κατασκευάζω την κατοπτρική

$$y_{e,(s)}(m) = y((-m))_{2N}$$

Παράδειγμα

$$y_{e,(s)}(0) = y((0))_N = 1$$

$$y_{e,(s)}(1) = 0$$

$$y_{e,(s)}(2) = 0$$

$$y_{e,(s)}(3) = 0$$

$$y_{e,(s)}(4) = 0$$

$$y_{e,(s)}(5) = 2$$

$$y_{e,(s)}(6) = 0.5$$

$$y_{e,(s)}(7) = 1$$

Παράδειγμα

Κατασκευάζω την κατοπτρική της

$$x \to \{1,0,2.5,1.5,0,0,0,0\}$$

$$y_{(s)} \to \{1,0,0,0,0,2,0.5,1\}$$

$$z[0] = 1$$

$$x \to \{1,0,2.5,1.5,0,0,0,0\}$$

$$y_{(s)} \to \{1,1,0,0,0,0,2,0.5\}$$

$$z[1] = 1$$

$$x \to \{1,0,2.5,1.5,0,0,0,0\}$$

$$y_{(s)} \to \{0.5,1,1,0,0,0,0,2\}$$

$$z[2] = 0.5 + 2.5 = 3$$

Παράδειγμα

Κατασκευάζω την κατοπτρική της

$$x_e \rightarrow \{1,0,2.5,1.5,0,0,0,0\}$$

$$y_{e,(s)} \rightarrow \{2,0.5,1,1,0,0,0,0\}$$

$$z[3] = 2 + 2.5 + 1 = 5.5$$

$$x_e \rightarrow \{1,0,2.5,1.5,0,0,0,0\}$$

$$y_{e,(s)} \rightarrow \{0,2,0.5,1,1,0,0,0\}$$

$$z[4] = 0.5 \times 2.5 + 1.5 = 2.75$$

$$x_e \rightarrow \{1,0,2.5,1.5,0,0,0,0\}$$

$$y_{e,(s)} \rightarrow \{0,0,2,0.5,1,1,0,0\}$$

$$z[5] = 5 + 1.5 \times 0.5 = 5.75$$

Παράδειγμα

Κατασκευάζω την κατοπτρική της

$$x_e \to \{1,0,2.5,1.5,0,0,0,0\}$$

$$y_{e,(s)} \to \{0,0,0,2,0.5,1,1,0\}$$

$$z[6] = 3$$

$$x_e \to \{1,0,2.5,1.5,0,0,0,0\}$$

$$y_{e,(s)} \to \{0,0,0,0,2,0.5,1,1\}$$

$$z[7] = 0$$

Οι τιμές του z δίνουν τη γραμμική συνέλιξη των x και y!!!

Παράδειγμα

- Έστω x[0]=1.5, x[1]=2.3+j0.5, x[2]=3.1-j5.6, x[3]=0.7+j2.7, x[4]=4.6 και γνωρίζεται ότι N=8 και ότι όλες οι τιμές του DFT είναι πραγματικοί αριθμοί. Ποιες οι 3 τιμές του x που λείπουν;

<u>Λύση</u>

Από τον αντίστροιφο DFT ισχύει, $$x[n] = \frac{1}{N}\sum_{n=0}^{N-1} X(k)(W_N)^{-nk}$$

και

$$x[N-n] = \frac{1}{N}\sum_{k=0}^{N-1} X(k)(W_N)^{-(N-n)k} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)(W_N)^{nk}$$

$$x[n] = x^*[N-n]$$

Παράδειγμα

- Έστω x[0]=1.5, x[1]=2.3+j0.5, x[2]=3.1-j5.6, x[3]=0.7+j2.7, x[4]=4.6 και γνωρίζεται ότι N=8 και ότι όλες οι τιμές του DFT είναι πραγματικοί αριθμοί. Ποιες οι 3 τιμές του x που λείπουν;

<u>Λύση</u>

Άρα

$$x[n] = x^*[N-n]$$

$$x[5] = x^*[8-5] = x^*[3] = 0.7 - j2.7$$

$$x[6] = x^*[8-6] = x^*[2] = 3.1 + j5.6$$

$$x[7] = x^*[8-7] = x^*[1] = 2.3 - j0.5$$

Φυσικα το x[4] είναι πραγματικός αριθμός!!! (γιατί??)

Παράδειγμα

This problem is a simple example of the use of superposition. Suppose that a discrete-time linear system has outputs $y[n]$ for the given inputs $x[n]$ as shown in Figure P4.1-1.

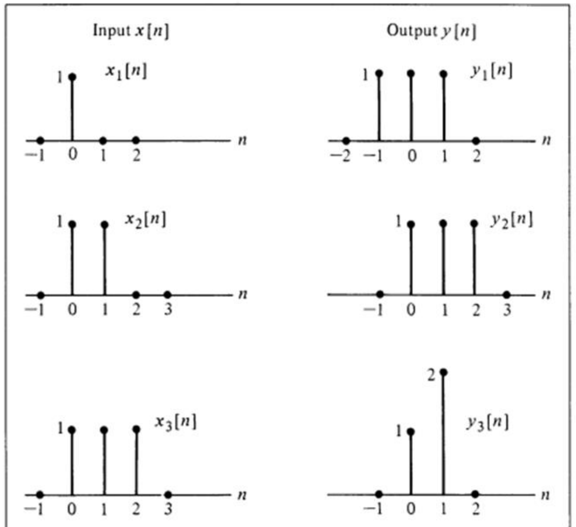

Figure P4.1-1

Determine the response $y_4[n]$ when the input is as shown in Figure P4.1-2.

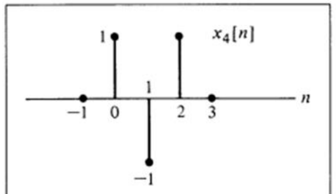

Figure P4.1-2

(a) Express $x_4[n]$ as a linear combination of $x_1[n]$, $x_2[n]$, and $x_3[n]$.

(b) Using the fact that the system is linear, determine $y_4[n]$, the response to $x_4[n]$.

(c) From the input-output pairs in Figure P4.1-1, determine whether the system is time-invariant.

The given input in Figure S4.1-1 can be expressed as linear combinations of $x_1[n]$, $x_2[n]$, $x_3[n]$.

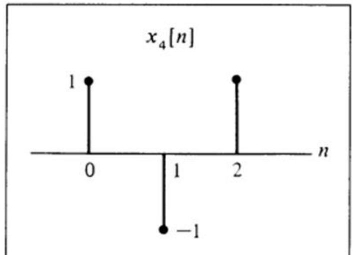

Figure S4.1-1

(a) $x_4[n] = 2x_1[n] - 2x_2[n] + x_3[n]$

(b) Using superposition, $y_4[n] = 2y_1[n] - 2y_2[n] + y_3[n]$, shown in Figure S4.1-2.

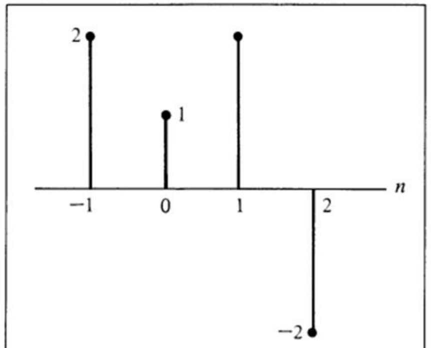

Figure S4.1-2

(c) The system is not time-invariant because an input $x_1[n] + x_1[n-1]$ does not produce an output $y_1[n] + y_1[n-1]$. The input $x_1[n] + x_1[n-1]$ is $x_1[n] + x_1[n-1] = x_2[n]$ (shown in Figure S4.1-3), which we are told produces $y_2[n]$. Since $y_2[n] \neq y_1[n] + y_1[n-1]$, this system is not time-invariant.

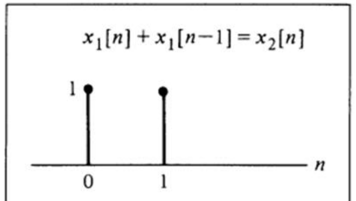

Figure S4.1-3

Παράδειγμα

Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following two cases.

(a)

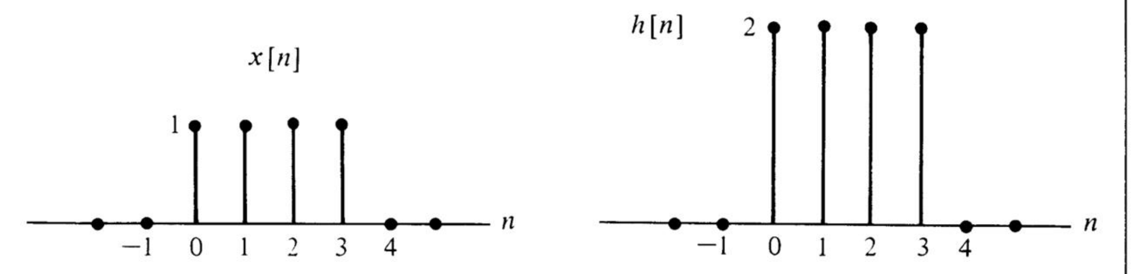

Figure P4.2-1

(b)

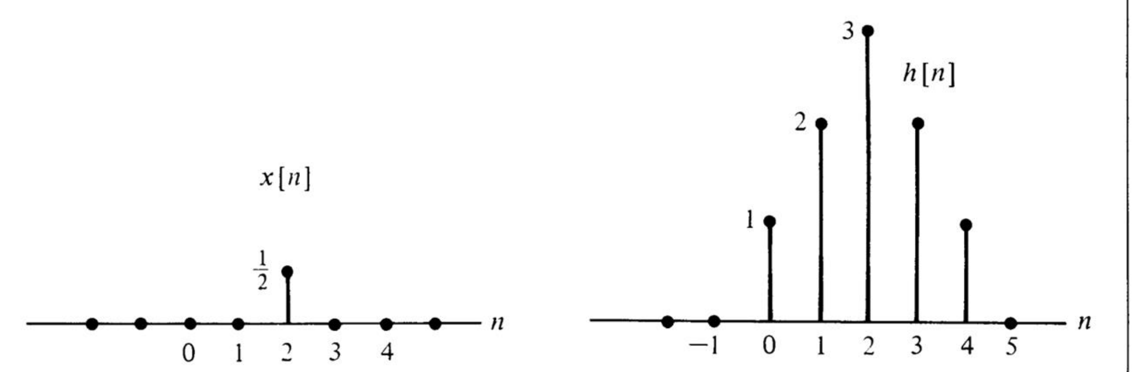

Figure P4.2-2

The required convolutions are most easily done graphically by reflecting $x[n]$ about the origin and shifting the reflected signal.

(a) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-1.

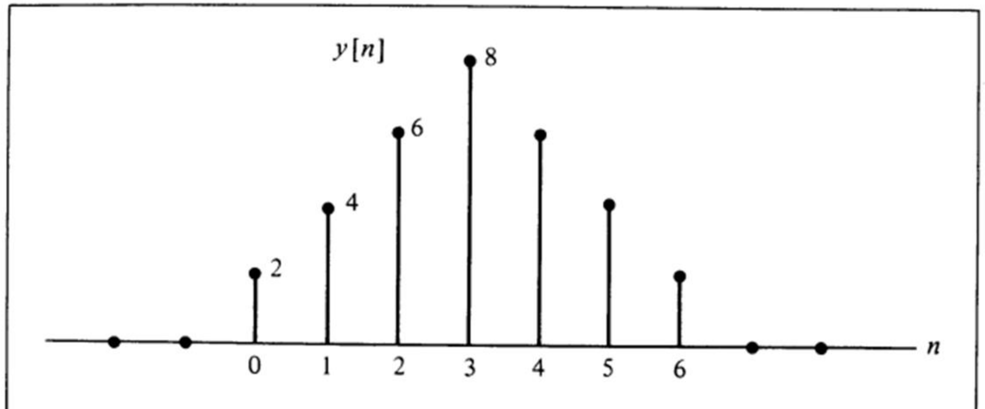

Figure S4.2-1

(b) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-2.

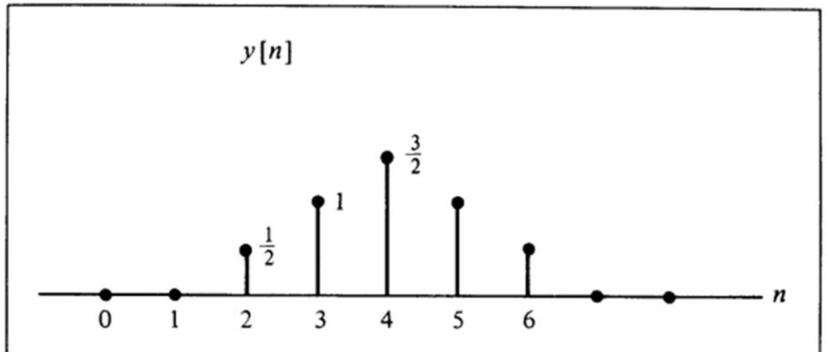

Figure S4.2-2

Notice that $y[n]$ is a shifted and scaled version of $h[n]$.